JEE (Advanced)-2018
Paper-I
MATHEMATICS

SECTION 1 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+4** If only (all) the correct option(s) is (are) chosen.
Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.
Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : **−2** In all other cases.

- **For Example:** If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option) ,without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks.

1. For a non-zero complex number z, let arg(z) denote the principal argument with $-\pi < \arg(z) \le \pi$. Then, which of the following statement(s) is (are) FALSE?

(A) $Arg(-1-i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$

(B) The function $f : R \to (-\pi, \pi]$, defined by f(t) = arg (-1 + it) for all $t \in R$, is continuous at all points of R, where $i = \sqrt{-1}$

(C) For any two non-zero complex numbers z_1 and z_2, $\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$ is an integer multiple of 2π

(D) For any three given distinct complex numbers z_1, z_2 and z_3, the locus of the point z satisfying the condition $\arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi$, lies on straight line.

Ans. (ABD)

Sol. (A) $Arg(-1-i) = -\frac{3\pi}{4}$

(B) f(t) = Arg(-1 + it) $\begin{bmatrix} \pi - \tan^{-1} t & t \ge 0 \\ -(\pi + \tan^{-1} t) & t < 0 \end{bmatrix}$

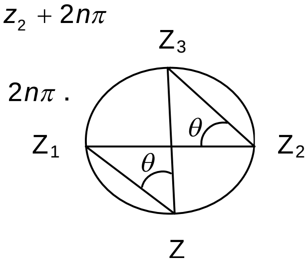
If is discontinuous at t = 0.

(C) $Arg\left(\frac{z_1}{z_2}\right) - Arg\, z_1 + Arg\, z_2$

$$Arg\left(\frac{z_1}{z}\right) = Arg\, z_1 - Arg\, z_2 + 2n\pi$$

so the expression becomes $2n\pi$.

(D) $Arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi$



If is circle

2. In a triangle PQR, let $\angle PQR = 30^0$ and the sides PQ and QR have lengths $10\sqrt{3}$ and 10, respectively. Then, which of the following statement(s) is (are) TRUE?

(A) $\angle QPR = 45^0$

(B) The area of the triangle PQR is $25\sqrt{3}$ and $\angle QRP = 120^0$.

(C) The radius of the incircle of the triangle PQR is $10\sqrt{3} - 15$

(D) The area of the circumcircle of the triangle PQR is 100π.

Ans. (BCD)

Sol.

$$\cos Q = \frac{100+300-(PR)^2}{2.10.10\sqrt{3}} \Rightarrow \frac{\sqrt{3}}{2} = \frac{100+300-(PR)^2}{2.10.10\sqrt{3}}$$

$300 = 400 - (PR)^2 \quad \Rightarrow PR = 10$

$$\Delta = \frac{1}{2}(PQ)(QR)\sin Q = \frac{1}{2}\,10.10\sqrt{3} \times \frac{1}{2} = 25\sqrt{3}$$

$$r = \frac{\Delta}{s}\ \frac{25\sqrt{3}\times 2}{\left(20+10\sqrt{3}\right)} = \frac{50\sqrt{3}}{20+10\sqrt{3}} = \frac{5\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= 5\left(2\sqrt{3}-3\right) = 10\sqrt{3} - 15$$

by sine rule $\frac{10\sqrt{3}}{\sin R} = \frac{10}{\sin Q} \quad \Rightarrow \angle R = 30^0$

2(circumradius) = $\frac{PR}{\sin Q} = \frac{10}{1/2}$

$\Rightarrow$ circumradius = 10

Hence area of circumcircle $= \pi R^2 = 100\pi$

3. Let P_1 : 2x + y – z = 3 and P_2: x + 2y + z= 2 be two planes. Then, which of the following statement(s) is (are) TRUE?

(A) The line of intersection of P_1 and P_2 has direction ratios 1, 2, -1

(B) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2.

(C) The acute angle between P_1 and P_2 is 60^0.

(D) If P_3 is the plane passing through the point (4, 2, -2) and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point (2, 1, 1) from the plane P_3 is $\frac{2}{\sqrt{3}}$.

Ans.(CD)

Sol. Direction ratio of common line is n_1 x n_2

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = \hat{i}(3) - \hat{j}(3) + \hat{k}(3) = 3\left(\hat{i} - \hat{j} + k\right)$$

(B) $\frac{x-4/3}{3} = \frac{y-1/3}{-3} = \frac{z}{3}$

This is || to the line of intersection

(C) $\cos\theta = \frac{x_i x_2}{|x_1||x_2|} = \frac{2+2-1}{\sqrt{6}\sqrt{6}} = \frac{3}{6} = \frac{1}{2} \quad \Rightarrow \theta = \frac{\pi}{3}$

(D) P_3 : x – y + z = λ satisfy (4, 2, -2)

4 – 2 – 2 = $\lambda \quad \Rightarrow$ x – y + z = 0

$(2, 1, 1) \perp \Rightarrow \left|\frac{2-1+1}{\sqrt{3}}\right| \quad \Rightarrow \frac{2}{\sqrt{3}}$

4. For every twice differentiable function f : R → [-2, 2] with $(f(0))^2 + (f'(0))^2 = 85$, which of the following statement(s) is (are) TRUE?

(A) There exist r, s $\in$ R, where r < s, such that f is one-one on the open interval (r,s)

(B) There exists $x_0 \in (-4, 0)$ such that $|f'(x_0)| \le 1$

(C) $\lim_{x\to\infty} f(x) = 1$

(D) There exists $\alpha \in (-4, 4)$ such that $f(\alpha) + f''(\alpha) = 0$ and $f'(\alpha) \neq 0$.

Ans (ABD)

Sol. $f^2(0) + (f'(0))^2 = 85 \qquad$ f : R → [-2, 2]

(A) This is true of every continuous function

(B) $f'(c) = \dfrac{f(-4) - f(0)}{-4-0}$

$|f'(c)| = \left|\dfrac{f(-4) - f(0)}{4}\right|$ $\qquad -2 \le f(-4) \le 2$

$-2 \le f(0) \le 2$

$\overline{-4 \le f(-4) - f(0)} \le 4$ This $|f'(c)| \le 1$

(C) $\lim\limits_{x\to\infty} f(x) = 1$

Note f(x) should have a bound ∞ which can be concluded by considering

$f(x) = 2\sin\left(\dfrac{\sqrt{85}\,x}{2}\right)$

$f'(x) = \sqrt{85}\cos\left(\dfrac{\sqrt{85}\,x}{2}\right)$, $f^2(0) + (f'(0)^2) = 85$

and $\lim\limits_{x\to\infty} f(x)$ does not exist.

(D) Consider $H(x) = f^2(x) + (f'(x)^2$ $H(0) = 85$

By (B) choice there exists some x_0 such that $(f'(x_0))^2 \le 1$ for some x_0 in (-4, 0) hence $H(x_0) = f^2(x_0) + (f'(x_0))^2 \le 4 + 1$ $H(x_0) \le 5$.

Hence let $p \in (-4, 0)$ for which H(p) = 5

(note that we have considered p as largest such negative number) similarly let q be smallest positive number $\in (0, 4)$ such that H(q) = 5. Hence By Rolle's theorem is (p, q).

H'(c) = 0 for some $c \in (-4, 4)$ and since H(x) is greater than 5 as we move from x = p to x = q and $f^2(x) \le 4$.

$\Rightarrow (f'(x))^2 \ge 1$ in (p, q)

Thus $H'(c) = 0 \Rightarrow f'f + f'f'' = 0$

so f + f" = 0 and $f' \ne 0$.

5. Let f : R →R and g : R → R be two non-constant differentiable functions. If $f'(x) = (e^{(f(x)-g(x))})\, g'(x)$ for all $x \in R$, and f(1) = g(2) = 1, then which of the following statement(s) is (are) TRUE?

(A) $f(2) < 1 - \log_e 2$

(B) $f(2) > 1 - \log_e 2$

(C) $g(1) > 1 - \log_e 2$

(D) $g(1) < 1 - \log_e 2$

Ans. (BC)

Sol. $f'(x) = e^{f(x)-g(x)}\, g'(x)$: f(1) = g(2) = 1

$e^{-f(x)} = e^{-g(x)} + c$

$e^{-f(x)} \cdot f'(x) = e^{-g(x)} \cdot g'(x)$

$\int d\left(e^{-f(x)}\right) = \int d\left(e^{-g(x)}\right)$

$e^{-f(x)} = e^{-g(x)} + c$

$x = 1$ $\dfrac{1}{e} = e^{-g(1)} + c$ $\qquad x = 2$ $e^{-f(2)} = \dfrac{1}{e} + c$

$\therefore$ g(1) > 1 – log 2

$e^{-f(2)} = 2e^{-1} - e^{-g(1)}$

$e^{-f(2)} = 2e^{-1} - e^{-g(1)}$ $\qquad$ f(2) > 1 – log 2

$e^{-1} - e^{-f(2)} = e^{-g(1)} - e^{-1} \Rightarrow e^{-g(1)} + e^{-f(2)} = 2e^{-1}$

$e^{-g(1)} < 2e^{-1}$, -g(1) < log 2 – 1

6. Let $f : [0, \infty) \to R$ be a continuous function such that $f(x) = 1 - 2x + \int_0^x e^{x-1} f(t)\,dt$ for all $x \in [0, \infty)$. Then, which of the following statement(s) is (are) TRUE?

(A) The curve y = f(x) passes through the point (1, 2)

(B) The curve y = f(x) passes through the point (2, -1)

(C) The area of the region

$\{(x, y) \in [0, 1] \times R : f(x) \le y \le \sqrt{1-x^2}$ is $\dfrac{\pi - 2}{4}$

(D) The area of the region

$\{(x, y) \in [0, 1] \times R : f(x) \le y \le \sqrt{1-x^2}$ is $\dfrac{\pi - 1}{4}$

Ans (B,C)

Sol. $f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)\,dt$

$\Rightarrow f(x).e^{-x} = (1-2x).e^{-x} + \int_0^x e^{-1} f(t)\,dt$

$\Rightarrow f'(x)e^{-x} - e^{-x}.f(x) = -2.e^{-x} - (1-2x).e^{-x} + e^{-x}.f(x)$

$\Rightarrow f'(x) - 2f(x) = (2x-3)$ I.F. = e^{-2x}

$\therefore y.e^{-2x} = \int (2x-3).e^{-2x}\,dx$

$\Rightarrow y.e^{-2x} = (2x-3).\dfrac{e^{-2x}}{-2} - 2\int \dfrac{e^{-2x}}{-2}\,dx$

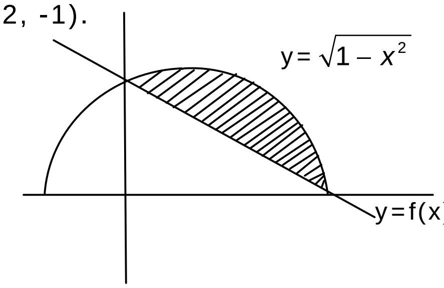
$\Rightarrow y.e^{-2x} = -\dfrac{(2x-3)e^{-2x}}{2} - \dfrac{e^{-2x}}{2} + c$

$\Rightarrow y.e^{-2x} = \dfrac{-(2x-3)-1}{2} + c.e^{2x}$

$\Rightarrow y = (1-x) + c.e^{2x}$ put x = 0,

1 = 1 + c $\Rightarrow c = 0$

$\therefore$ y = 1 – x which passes through point (2, -1).



Now, required area $= \dfrac{1}{4}.\pi.(1)^2 - \dfrac{1}{2}.1.1 = \dfrac{\pi}{4} - \dfrac{1}{2}$

SECTION – 2 : (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions. The answer to each question is **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, –0.33, –0.30,,30.27, –127.30) using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme :
Full Marks : **+3** If ONLY the correct numerical value is entered as answer. Zero Marks : **0** In all other cases.

7. The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times x\left(\sqrt{7}\right)^{\frac{1}{\log_4 7}}$ is _____

Ans (8)

Sol. $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times \left(\sqrt{7}\right)^{\log_7 4}$

$(\log_2 9)^{2\log_{(\log_2 9)} 2}.(2) = 4.2 = 8$

8. The number of 5 digit numbers which are divisible by 4, with digits from the set {1, 2, 3, 4, 5} and the repetition of digits is allowed, is _____

Ans. (625)

Sol. Last two digits are 12, 32, 24, 52, 44

Number of numbers = $5 \times 5 \times 5 \times 5 = 625$

9. Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, ... and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, ... Then, the number of elements in the $X \cup Y$ is ______.

Sol. (3748)

P = {1, 6, 11, ...}

Q = {9, 16, 23, ...}

Common terms : 16, 51, 86

T_p = 16 + (p – 1)35 = 35p – 19 $\le$ 10086

$\Rightarrow p \le 288.7$

$\therefore n(p \cup Q) = n(P) + n(Q) - n(P \cap Q)$

= 2018 + 2018 – 288 = 3748

10. The number of real solutions of the equation

$\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i\right) = \frac{\pi}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty}\left(-\frac{x}{2}\right)^i - \sum_{i=1}^{\infty}(-x)^i\right)$

lying in the interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$ is ______.

(Here, the inverse trigonometric function $\sin^{-1}x$ and $\cos^{-1}x$ assume values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$ respectively).

Ans. (2)

Sol.

$$\sin^{-1}\left(\sum_{i=1}^{\infty}x^{i+1}-x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i\right)=\frac{\pi}{2}-\cos^{-1}\left(\sum_{i=1}^{\infty}\left(-\frac{x}{2}\right)^i-\sum_{i=1}^{\infty}(-x)^i\right)$$

$$\left(\frac{x^2}{1-x}-x-\frac{\frac{x}{2}}{1-\frac{x}{2}}\right)=\frac{x}{1+x}+\frac{\left(-\frac{x}{2}\right)}{1+\frac{x}{2}}$$

$$\frac{x^2}{1-x}-\frac{x^2}{2-x}=\frac{x}{1+x}-\frac{x}{2+x}$$

$$\frac{x^2}{1-x}-\frac{x}{1+x}=\frac{x^2}{2-x}-\frac{x}{2+x}$$

$$\frac{x(1+x)-(1-x)}{1-x^2}=\frac{2x+x^2-2+x}{4-x^2} \quad \text{or } x=0$$

$$\frac{x^2+2x+1}{1-x^2}=\frac{x^2+3x-2}{4-x^2}$$

$\Rightarrow x^3+2x^2+5x-2=0$

Let $f(x)=x^3+2x^2+5x-2$ f'x) > 0

f(0)=-2 and f(1/2)=9/8 so one root in $\left(0,\frac{1}{2}\right)$

$\Rightarrow$ 2 roots

11. For each positive integer n, let $y_n=\frac{1}{n}\left((n+1)(n+2)\ldots.(n+n)\right)^{1/n}$. For $x\in R$, let [x] be the greatest integer less than or equal to x. If $\lim\limits_{n\to\infty}y_n=L$, then the value of [L] is ______

Ans (1)

Sol. $y_n=\left(\frac{n+1}{n}\frac{n+2}{n}\ldots.\frac{n+n}{n}\right)^{\frac{1}{n}}$

$$\log L=\lim_{x\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(1+\frac{r}{n}\right)$$

$$=\int_0^1\log(1+x)\,dx=\int_1^2\log x\,dx=\left|x\log x-x\right|_1^2=2\log 2=\log\frac{4}{e}$$

$\Rightarrow L=\frac{4}{e} \quad \Rightarrow [L]=1$

12. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a}.\vec{b}=0$. For some $x, y\in R$, let $\vec{c}=x\vec{a}+y\vec{b}+\left(\vec{a}\times\vec{b}\right)$. If $|\vec{c}|=2$ and the vector $\vec{c}$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2\alpha$ ____

Ans. (3)

Sol. $\vec{c}=x\vec{a}+y\vec{b}+\vec{a}\times\vec{b}$ & $\vec{a}.\vec{b}=0$

$\vec{a}\,\hat{}\,\vec{c}=\vec{b}\,\hat{}\,\vec{c}=\alpha$

$\vec{c}.\vec{a}=\vec{c}.\vec{b}=2\cos\alpha \quad \Rightarrow x=y=2\cos\alpha$

$\left|\vec{c}\right|^2=x^2+y^2+\left|\vec{a}\times\vec{b}\right|^2=2\left(4\cos^2\alpha\right)+1-0$

$4=8\cos^2\alpha+1 \quad \Rightarrow 8\cos^2\alpha=3$

13. Let a, b, c be three non-zero real numbers such that the equation $\sqrt{3}a\cos x+2b\sin x=c$, $x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, has two distinct real roots α *and* β with $\alpha+\beta=\frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is _____

Ans. (0.5)

Sol. $\sqrt{3}a\cos x+2b\sin x=c \quad x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$,

$\sqrt{3}a\left(\frac{1-t^2}{1+t^2}\right)+2b\left(\frac{2t}{1+t^2}\right)=c$, where $t=\tan\frac{x}{2}$

$\sqrt{3}a\left(1-t^2\right)+4bt=c\left(1+t^2\right)$

$t^2\left(c+\sqrt{3}a\right)-4bt+c-\sqrt{3}a=0$

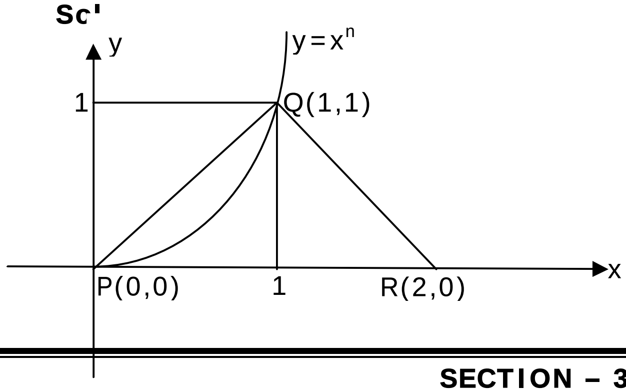
$\frac{\alpha+\beta}{2}=\frac{\pi}{6}$, $\tan\left(\frac{\alpha+\beta}{2}\right)=\frac{1}{\sqrt{3}} \quad \Rightarrow \frac{t_1+t_2}{1-t_1t_2}=\frac{1}{\sqrt{3}}$

$\Rightarrow \frac{4b}{c+\sqrt{3}a-c+\sqrt{3}a}=\frac{1}{\sqrt{3}} \qquad \frac{b}{a}=\frac{1}{2}$

14. A farmer F_1 has a land in the shape of a triangle with vertices at P(0, 0), Q(1, 1) and R(2, 0). From this land, a neighboring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y=x^n$ (n > 1). If the area of the region taken away by the farmer F_2 is exactly 30% of the area of ΔPQR, then the value of n is

Ans. (4)

Sc'



$$\int_0^1 \left(x - x^n\right) dx = \frac{3}{10}\left(\frac{1}{2} \times 2 \times 1\right)$$

$$\Rightarrow \left|\frac{x^2}{2} - \frac{x^{n+1}}{n+1}\right|_0^1 = \frac{3}{10} \qquad \Rightarrow \frac{1}{2} - \frac{1}{n+1} = \frac{3}{10}$$

$$\Rightarrow \frac{1}{n+1} = \frac{1}{2} - \frac{3}{10} = \frac{1}{5} \qquad \Rightarrow n = 4$$

SECTION – 3 : (Maximum Marks : 12)

- This section contains **TWO (02)** paragraphs. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options corresponds to the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : **+3** If ONLY the correct option is chosen.

Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : **–1** In all other cases.

PARAGRAPH "X"

Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$.

(There are two questions based on PARAGRAPH "X", the question given below is one of them)

15. Let E_1E_2 and F_1F_2 be the chords of S passing through the point $P_0(1, 1)$ and parallel to the x-axis and the y-axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slope – 1. Let the tangents to S at E_1 and E_2 meet at E_3, the tangents to S at F_1 and F_2 meet at F_3, and the tangents to S at G_1 and G_2 meet at G_3. Then, then, the points E_3, F_3 and G_3 lie on the curve.

(a) $x + y = 4$

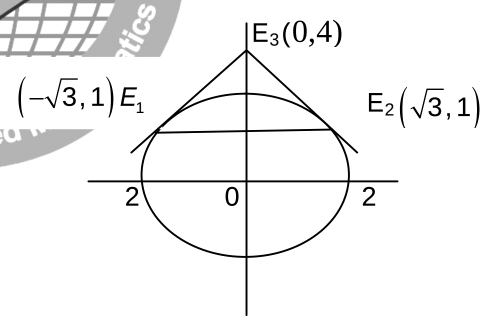
(b) $(x - 4)^2 + (y - 4)^2 = 16$

(c) $(x - 4)(y - 4) = 4$

(d) $xy = 4$

Sol. (A)

Tangent at E_1 and E_2 are $-\sqrt{3}x + y = 4$ and $\sqrt{3}x + y = 4$. They intersect at E_3 (0, 4)



$F_1\left(1, \sqrt{3}\right), F_2\left(1, -\sqrt{3}\right), F_3\left(4, 0\right)$

$G_1\left(0, 2\right),\ G_2\left(2, 0\right),\ G_3\left(2, 2\right)$

E_3, F_3, G_3 lie on line $x + y = 4$

PARAGRAPH "X"

Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$.

(There are two questions based on PARAGRAPH "X", the question given below is one of them)

16. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve

(A) $(x + y)^2 = 3xy$ (B) $x^{2/3} + y^{2/3} = 2^{4/3}$

(C) $x^2 + y^2 = 2xy$ (D) $x^2 + y^2 = x^2y^2$

Ans.(D)

Sol. Let $P(2\cos\theta, 2\sin\theta)$

Tanget is $x\cos\theta + y\sin\theta = 2$

$M\left(\frac{2}{\cos\theta}, 0\right)$, $N\left(0, \frac{2}{\cos\theta}\right)$

$x = \frac{1}{\cos\theta}$ *and* $y = \frac{1}{\sin\theta}$

$\Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = 1 \qquad \Rightarrow x^2 + y^2 = x^2y^2$

PARAGRAPH "A"

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, i=1,2,3,4, 5. But, on the examination day, the five students are randomly allotted the five seats.

(There are two questions based on PARAGRAPH "A", the question given below:)

17. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1 and NONE of the remaining students gets the seat previously allotted to him/her, is

(A) $\frac{3}{40}$ (B) $\frac{1}{8}$ (C) $\frac{7}{40}$ (D) $\frac{1}{5}$

Sol. (A)

Probability $= \dfrac{4!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right)}{5!}$

$= \frac{9}{120} = \frac{3}{40}$

18. For i = 1, 2, 3, 4, let T_i denote the event that the students S_i and S_{i+1} do NOT sit adjacent to each other on the day of the examination. Then, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is

(A) $\frac{1}{15}$ (B) $\frac{1}{10}$ (C) $\frac{7}{60}$ (D) $\frac{1}{5}$

Ans.(C)

Sol. Total cases = 5! Favorable ways = 14

$\left.\begin{matrix} 1 & 3 & 5 & 2 & 4 \\ 1 & 4 & 2 & 5 & 3 \end{matrix}\right\} \to 2$

$5 \to 2$

2 4 1 .... → 2

2 5 3 1 4 →1

$4 \to 3$

$\left.\begin{matrix} 3 & 1 & 5 & 2 & 4 \\ 3 & 1 & 4 & 2 & 5 \end{matrix}\right\} \to 2$

$3 \quad 5 \quad \ldots \quad \ldots \quad \ldots\} \to 2 \quad = 14$

Probability $= \frac{14}{120}$